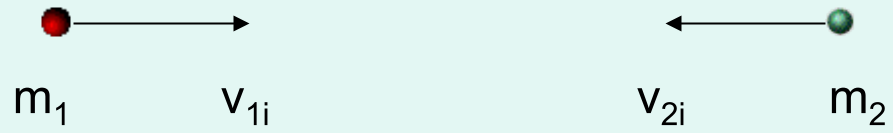
Elastic Collisions and Gravity

Steven Ball
LeTourneau University

Why Elastic Collisions?

- Most collisions in our day-to-day experience are far from elastic
- Most do not have an intuitive feel for elastic collisions.
- However, gravitational “collisions” are elastic and we can use this to redirect spacecraft and even boost spacecraft to higher velocities.

1-D Elastic Collisions



Conservation of linear momentum:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

Conservation of mechanical energy:

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

Conservative Forces

- Elastic Collisions require the force acting between the masses to be conservative
- In truly elastic collisions the masses must not touch! (e.g. magnetic bumpers)
- Collision = Transfer of Momentum

General Result:

$$v_{2f} - v_{1f} = -(v_{2i} - v_{1i})$$

Velocity difference remains the same (with a change in sign)

Demos with Pasco Dynamics Carts

For $m_1 = m_2$

$v_{1f} = v_{2i}$

$v_{2f} = v_{1i}$

Velocity Exchange!

Special Case: $m_1 = m_2$, $v_{2i} = 0$

$v_{1f} = 0$

$v_{2f} = v_{1i}$

1-D General Solution

$$v_{1f} = (m_1-m_2)/(m_1+m_2)\, v_{1i} + 2m_2/(m_1+m_2)\, v_{2i}$$

$$v_{2f} = 2m_1/(m_1+m_2)\, v_{1i} + (m_2-m_1)/(m_1+m_2)\, v_{2i}$$

Symmetric!

Large & Small Masses

For $m_2 >> m_1$

$$v_{1f} = -v_{1i} + 2\, v_{2i}$$

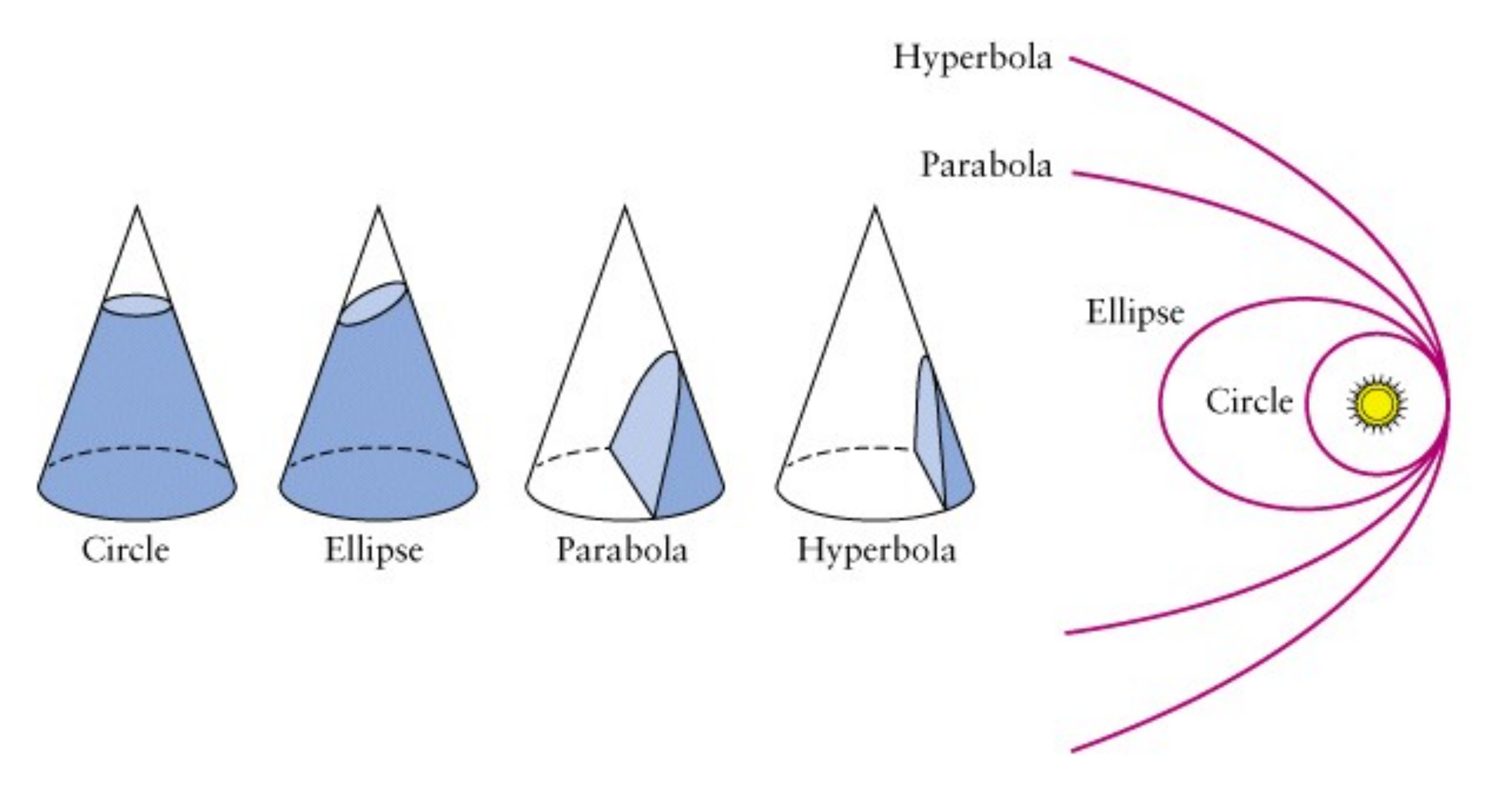
$$v_{2f} = v_{2i}$$

Special case: $v_{2i} = 0$

$$v_{1f} = -v_{1i}$$

Gravitational Trajectories: 2-D

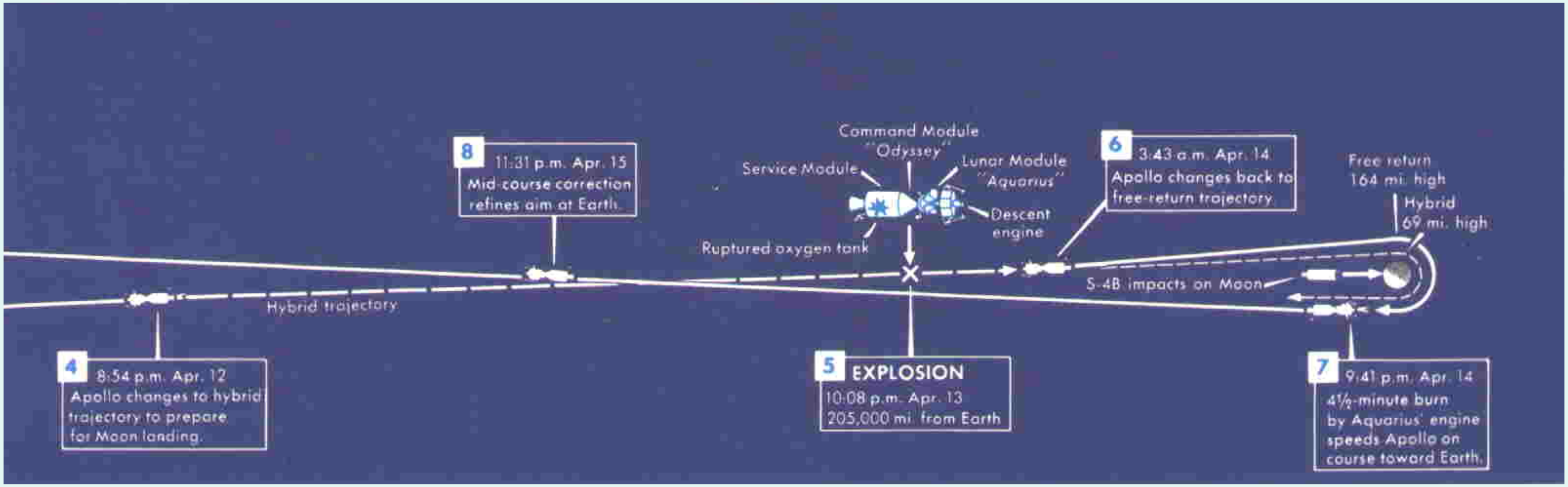
Conic Sections



Apollo 13 "Free Return" Trajectory

Mass of Moon $m_2 >> m_1$ Spacecraft

$$v_{1f} = -v_{1i}$$

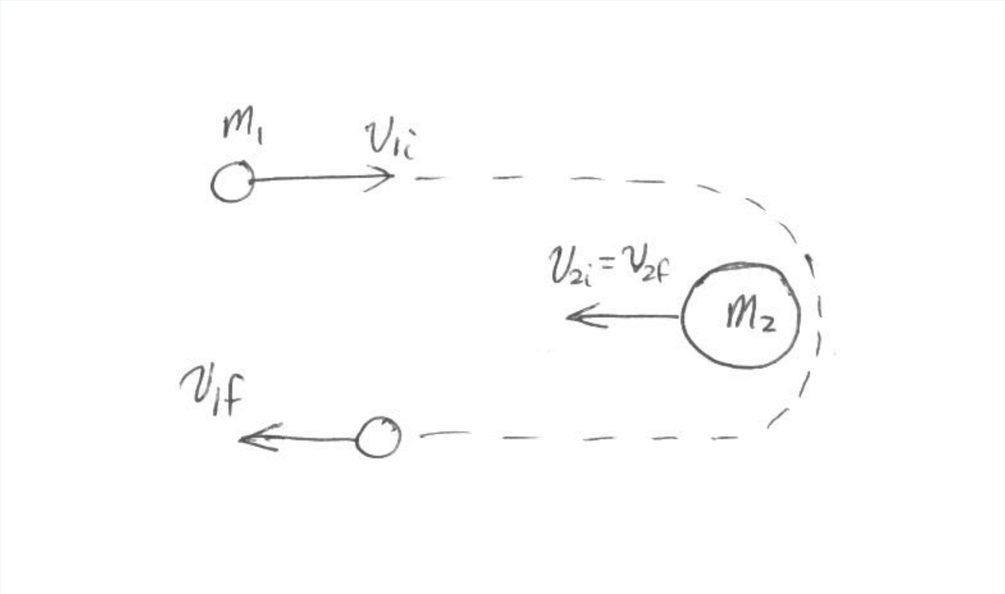


http://er.jsc.nasa.gov/seh/13index.htm

Gravitational Slingshot

Mass of Planet $m_2 >> m_1$ Spacecraft

$$v_{1f} = -v_{1i} + 2\,v_{2i}$$



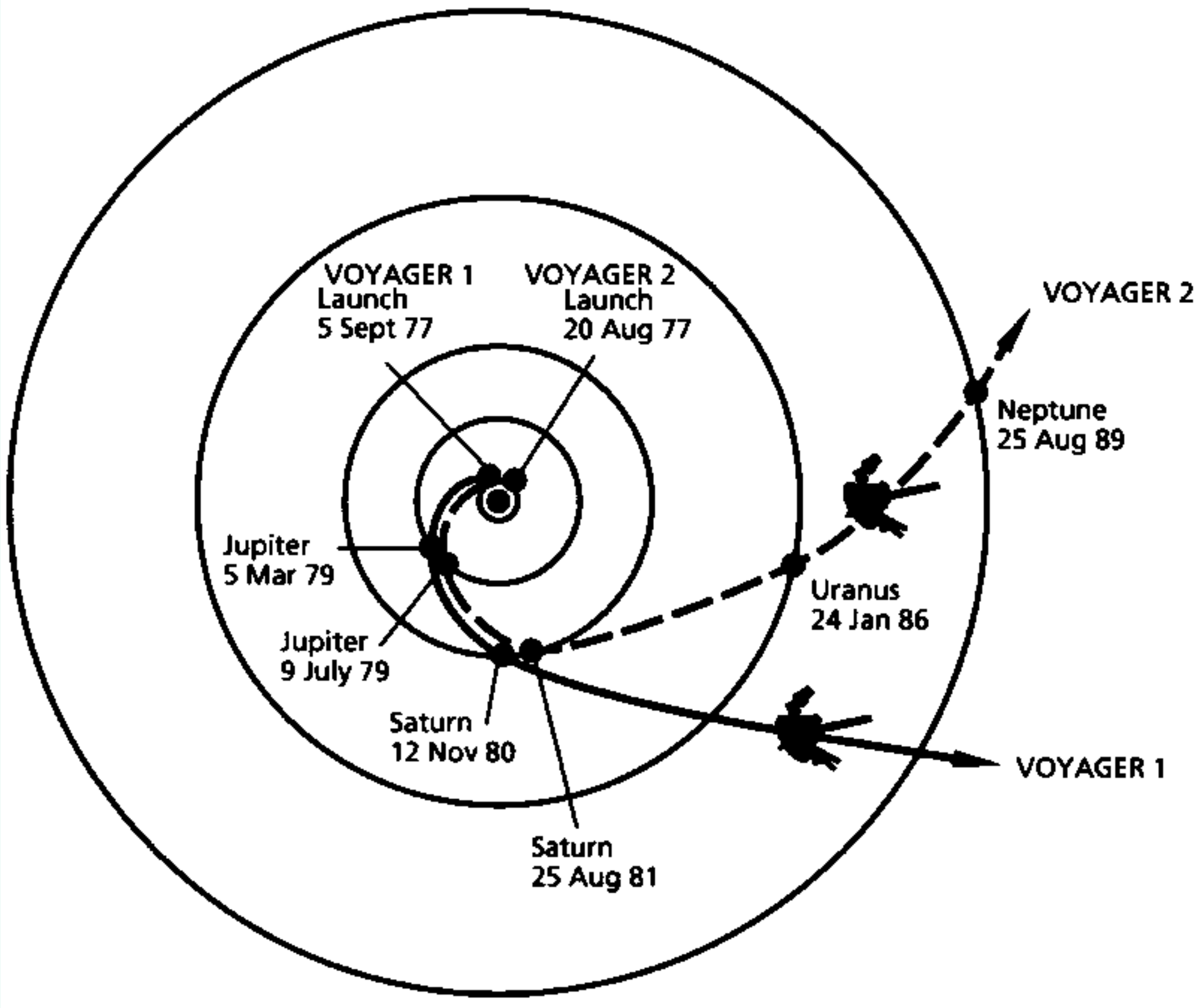
History of Gravitational Slingshot

- First proposed by Yuri Kondratyuk of the Ukraine in 1918-1919 to accelerate and decelerate spacecraft for interplanetary travel

- Shown to reduce fuel needed to propel a spacecraft into outer solar system by UCLA graduate student Michael Minovitch in 1961 during a summer internship at JPL

http://www.gravityassist.com/

Voyager Missions



Cassini Mission